Exercise: DELPHI method

New technology has been introduced to market. Delphi method was conducted in 2020 to predict implementation of new technology in XY company. The experts' answers to two of the questions in the first round were as follows:

Question no. 1: *When can we expect implementation of the new technology in our company?*							
Expert	2020	2021	2022	2023	2024	2025	2026
1			m				
2	o		m	p			
3		o		m		p	
4					m		
5		m					
6	o			m			p
7			o	m	p		
8	o		m	p			

Question no. 2: *When can we expect implementation of the new technology in 80% of companies in our industry?*							
Expert	2020	2021	2022	2023	2024	2025	2026
1							x
2					x		
3						x	
4							x
5	x						
6	x						
7		x					
8		x					

a) Based on the answers given, determine when those events can be expected;
b) Is the level of agreement higher for the first or for the second question?
c) Based on the answers, when can we expect the implementation of new technology in the organization with probability of 85%?
d) And with probability of 30%?

a)

Question no. 1: When can we expect implementation of the new technology in our company?							
In how many years	0	1	2	3	4	5	6
Expert	2020	2021	2022	2023	2024	2025	2026
1			m				
2	o		m	p			
3		o		m		p	
4					m		
5		m					
6	o			m			p
7			o	m	p		
8	o		m	p			

$$t_i = \frac{r_1 * o_i + r_2 * m_i + r_3 * p_i}{r_1 + r_2 + r_3} = \frac{1 * o_i + 4 * m_i + 1 * p_i}{6}$$ $r_1=1$, $r_2=4$, $r_3=1$ (PERT method)

$t_1 = 2$

$t_2 = \frac{1*0+4*2+1*3}{6} = 1{,}83$

$t_3 = 3$
$t_4 = 4$
$t_5 = 1$
$t_6 = 3$
$t_7 = 3$
$t_8 = 1{,}83$

$$t_n = \frac{1}{n}\sum_{i=1}^{k} t_i f_i$$

$$t_n = \frac{1}{n}\sum_{i=1}^{n} t_i = \frac{1}{8} * (2+1{,}83+3+4+1+3+3+1{,}83) = \mathbf{2{,}46}$$

(2020+2,46=2022,46)

Question no. 2: *When can we expect implementation of the new technology in 80% of companies in our industry?*							
In how many years?	0	1	2	3	4	5	6
Expert	2020	2021	2022	2023	2024	2025	2026
1							x
2					x		
3						x	
4							x
5	x						
6	x						
7		x					
8		x					

t_i	f_i	$t_i * f_i$	t_i^2	$t_i^2 * f_i$
0	2	0	0	0
1	2	2	1	2
2	0	0	4	0
3	0	0	9	0
4	1	4	16	16
5	1	5	25	25
6	2	12	36	72
Sum	8 experts	**23**	91	**115**

$$t_n = \frac{1}{n}\sum_{i=1}^{k} t_i f_i \quad = \frac{1}{8} * 23 = \mathbf{2{,}875}$$

(2020+2,875=2022, 875)

b) **Q1:**

$$\delta_i^2 = \frac{(p_i - o_i)^2}{r_4}$$ r4=36 (PERT method)

δ1= 0

δ2= $\frac{(3-0)^2}{36}$ =0,25

δ3=0,44

δ4=0

δ5=0

δ6=1

δ7=0,11

δ8=0,25

$$\delta_n^2 = \frac{1}{n}\left[\sum_{i=1}^{n}\delta_i^2 + \sum_{i=1}^{n}(t_i - t_n)^2\right] =$$

= $\frac{1}{8}$ * [(0+0,25+0,44+0+0+1+0,11+0,25) + (**2**- 2,46)² + (**1,83**- 2,46)² +(**3** - 2,46)² + (**4** - 2,46)² +
+ (**1**- 2,46)² + (**3** - 2,46)² + (**3** - 2,46)² +(**1,83** - 2,46)²]=
= $\frac{1}{8}$ * (2,05 +0,2116 + 0,3969 + 0,2916 + 2,3716 +2,1316 + 0,2916 + 0,2916 + 0,3969)=**1,05**

$\delta_n = \sqrt{\delta^2_n}$ **δn1=1,027**

Q2:

$\delta^2_n = \frac{1}{n}\sum_{i=1}^{k} f_i t_i^2 - t_n^2$ = $\frac{1}{8}$ *115 – 2,875² = **6,109** **δn2=2,47**

The level of agreement higher for the first question, as **δn1 <δn2**
(1,027) < (2,47).

c)

$$Y = \frac{t_i - t_n}{\delta_n}$$

Y > 0 => **P = 0.5 + Φ(Y)**

Y < 0 => **P** = 1 – [0.5 + Φ(Y)] **= 0.5 - Φ(Y)**

Q1:

t_n = 2,46
σ_n = 1,027

	t_i	$Y = \frac{t_i - t_n}{\delta_n}$	P = 0.5 ± Φ(Y)
2020.	0	-2,395	0.5 – Φ(-2,4) = 0,5-0,4920 =0,008
2021.	1	-1,42	0.5 – 0,4220 = 0,078
2022.	2	-0,45	0.5 – 0,1735 = 0,3265
2023.	3	0,53	0.5 + 0,2020 = 0,7020
2024.	4	1,499	0.5 + 0,4330 = 0,9330
2025.	5	2,47	0.5 + 0,4930 = 0,9930
2026.	6	3,45	0.5 + 0,4995 = 0,9995

P = 0,85 P > 0.5 => Y > 0 => P = 0.5 + Φ(Y)

0,85 = 0,5 + Φ(Y)
Φ(Y) = 0,85 – 0.5

Φ(Y) = 0,35 $\xrightarrow[\text{tab}]{\text{Laplas}}$ **Y = 1.04**

$$Y = \frac{t_i - t_n}{\delta_n}$$

$1{,}04 = \frac{t_i - 2{,}46}{1{,}027}$ => t_i = 1,04 * 1,027 + 2,46

t_i = 3,528

(2020+3,528=2023,528)

d)

$$Y = \frac{t_i - t_n}{\delta_n}$$

Y > 0 =>P = 0.5 + Φ(Y)
Y < 0 =>P = 0.5 - Φ(Y)

t_n = 2,46
σ_n = 1,027

P = 0,30 P < 0,5 => Y < 0 => P = 0,5 - Φ(Y)

0,30 = 0,5 – Φ(Y)
Φ(Y) = 0,5 – 0,3

Φ(Y) = 0,20 –Laplasova tablica–> **Y = - 0,53**

$$Y = \frac{t_i - t_n}{\delta_n}$$

$$-0.53 = \frac{t_i - 2,46}{1,027} \Rightarrow t_i = -0.53 * 1,027 + 2,46$$

t_i = 1,916

(2020+1,916=2021,916).